Diagnosing (and treating) ‘the maths problem’

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Overview

- **Diagnosis**
  - Anecdotal, non-scientific
  - Incontrovertible quantitative data
- **Symptoms**
  - Problems into Hons; not in isolation
- **Treatment**
  - A TDO is called for
  - A new course in problem solving

Maths

A-level paper 1988!

9205/2 **A JUNE**

MATHEMATICS

SYLLABUS C

PAPER 2

Wednesday **15 JUNE 1988** Morning 3 hours

Additional materials provided by the Syndicate:

1. Formulae (List MF5)
2. Mathematical Tables
3. Graph Paper (2 sheets)

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

INTERNATIONAL EXAMINATIONS

General Certificate of Education
Advanced Level

Maths

A-level paper 1988!

3 A particle P is projected, from a point O on horizontal ground, with speed V at an angle θ above the horizontal, where $\tan\theta = \frac{1}{3}$. The particle passes through the point with coordinates $(3a, \frac{3}{4}a)$ relative to horizontal and vertical axes at O in the plane of motion. Show that $V^2 = 20ga$. [5]

A particle Q is projected from O at the instant when P is moving horizontally. It strikes the ground at the same place and at the same instant as P. Show that the speed of projection of Q is $\sqrt{\left(\frac{145ga}{2}\right)}$ and find the tangent of the angle of projection. [9]

Maths

A-level paper 2007

7 An arrow is fired from a point A with a velocity of $25\,\mathrm{m\,s^{-1}}$, at an angle of 40° above the horizontal. The arrow hits a target at the point B which is at the same level as the point A, as shown in the diagram.

(a) State **two** assumptions that you should make in order to model the motion of the arrow. *(2 marks)*

(b) Show that the time that it takes for the arrow to travel from A to B is 3.28 seconds, correct to three significant figures. *(4 marks)*

(c) Find the distance between the points A and B. *(2 marks)*

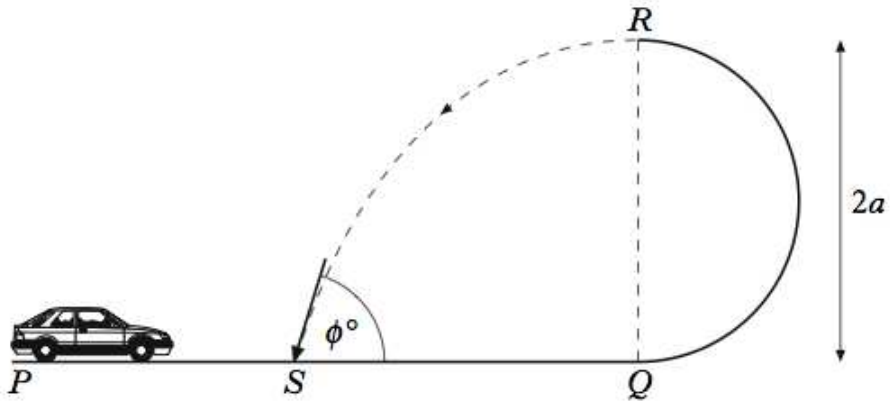
(d) State the magnitude and direction of the velocity of the arrow when it hits the target. *(2 marks)*

(e) Find the minimum speed of the arrow during its flight. *(2 marks)*

Maths

AH-level paper 2007

A9. The diagram below shows a smooth plastic track. The section PQ is horizontal and the section QR is semi-circular and in the same plane as PQ. The diameter QR is vertical and has length $2a$ metres.



A toy car is projected along PQ with speed $3\sqrt{ga}$ m s^{-1}. The car travels around the track to R, where it leaves the track horizontally, landing on PQ at the point S, where the angle between the car's trajectory and the line SQ is $\phi°$.

(*a*) Find the speed of the car at R, expressing your answer in the form $\sqrt{kga}$, where k is a constant. **3**

(*b*) Show that at R the car is in contact with the track. **2**

(*c*) Show that $SQ = 2\sqrt{5}a$ metres. **3**

(*d*) Find the exact value of $\tan \phi°$. **2**

Maths

AH-level paper 2007

A10. A rubber ball of mass m kg falls vertically into a tank of water. When the ball is x metres below the surface of the water and moving downwards with speed v m s^{-1}, the water exerts a resistive force of magnitude $2mv^2$ newtons and an upward buoyancy force of magnitude three times the weight of the ball.

(*a*) Show that the downward motion of the ball can be modelled by the differential equation

$$v\frac{dv}{dx} = -2(v^2 + g).$$ **2**

(*b*) The ball enters the water with speed U m s^{-1}. By solving the equation in (*a*), show that

$$v^2 + g = (U^2 + g)e^{-4x}.$$ **5**

(*c*) In the case when $U = 4{\cdot}9$, calculate, to the nearest centimetre, the greatest depth below the surface of the water reached by the ball. **3**

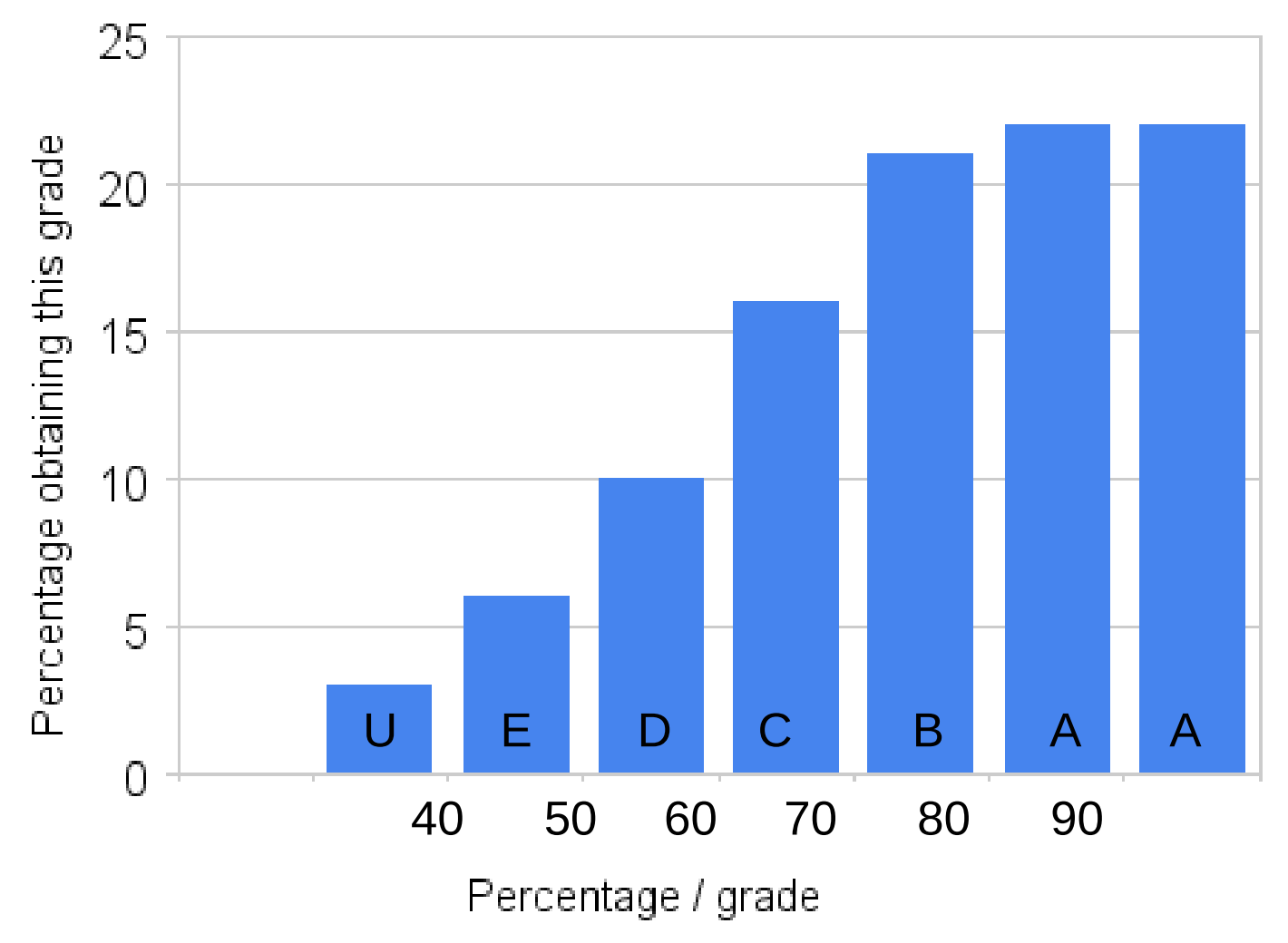
Maths

A-level results 2007

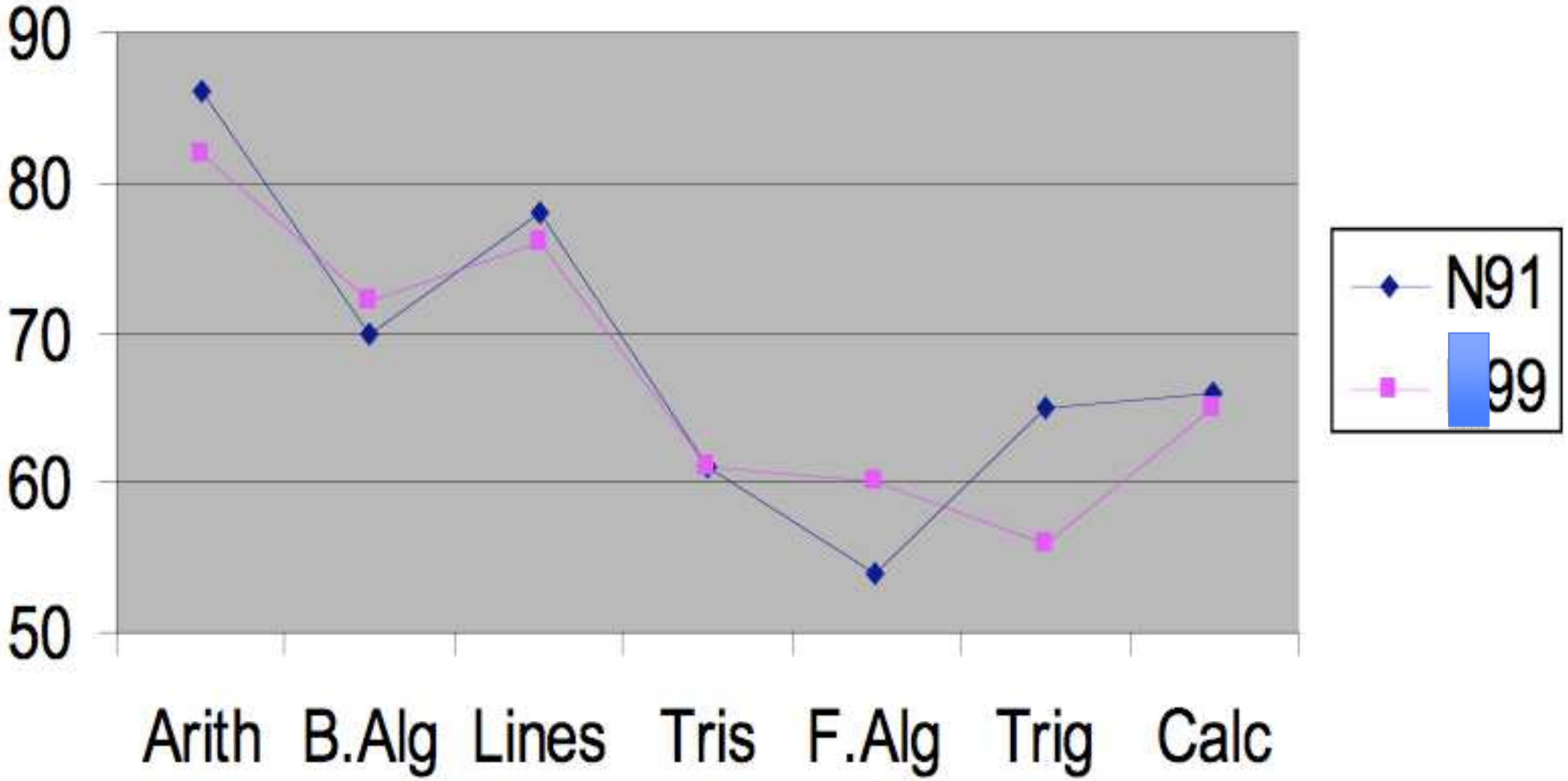
	Male	Female	Combined
Entries			60093
Grade A	42.6%	45.5%	43.7%
Grade B	20.8%	22.2%	21.4%
Grade C	15.8%	15.1%	15.6%
Grade D	10.9%	10.0%	10.4%
Grade E	6.4%	5.0%	5.9%
Grade U (Fail)	3.5%	2.2%	3.0%

Maths

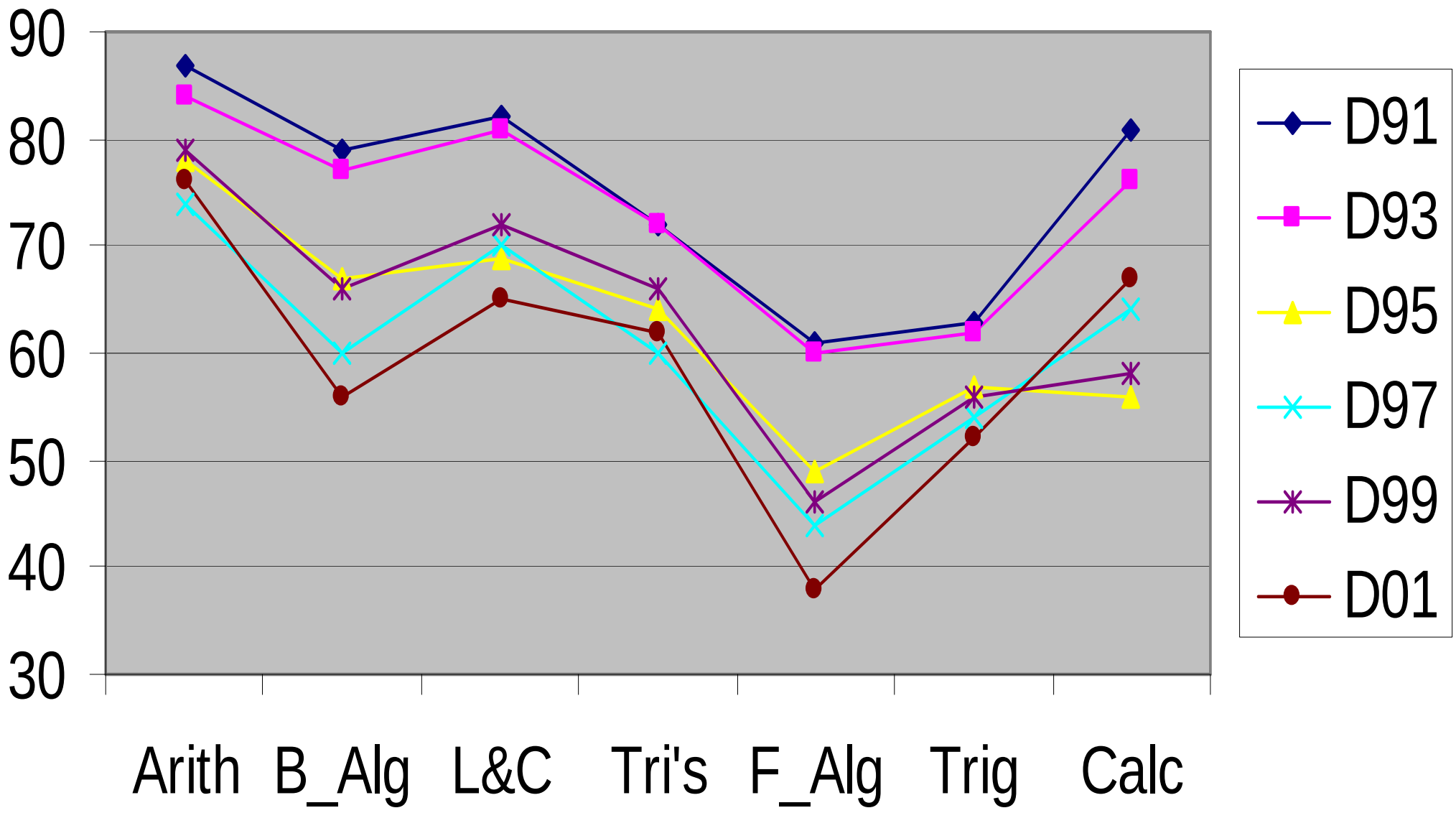
A-level results 2007

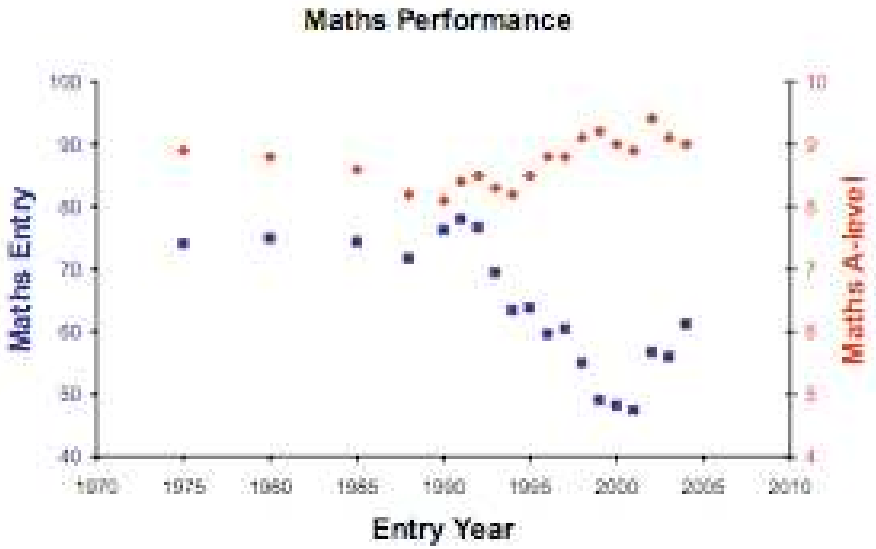


N91 vs [redacted]99 Diagnostic Test Results



Grade D Cohorts




Maths Performance
Maths Entry
Maths A-level
Entry Year
100
90
80
70
60
50
40
10
9
8
7
6
5
4
1970
1975
1980
1985
1990
1995
2000
2005
2010

Symptoms / manifestations

- **1st / 2nd year**
  - …getting by… in some cases (just)
- **Hons years**
  - Step change in difficulty
  - Wheels fall off….

Treatment

- Appointment of Teaching Development Officer

- New course, targetted at weakest students
- No new material
- No lectures, all studio-based
- Tutor-dense environment
- 800+ problems

Problem Solving in Physics

SCQF Level 8, U03739, PHY-2-PSIP

Monday 3rd December, 2007

9:30 - 12:30 a.m.

Confidence **Speed** **Mastery**

A.1 (i) Simplify as far as possible

$$\frac{st\left(1+\frac{s}{t}\right)\left(1-\frac{t}{s}\right)}{t^2-s^2}.$$

(ii) Extract h from the expression $v_0^2 = 2gR^2\left(\frac{1}{R} - \frac{1}{R+h}\right)$.

(iii) Factorise as far as possible $4\phi^3 - 4\phi^2 - \phi + 1$.

(iv) Find the first three non vanishing terms in the series expansion of $(\cos\theta)^{-1}$, where θ is small.

(v) Solve for x the equation $\ln 3 + \ln\left(\frac{x}{2} + \frac{1}{3}\right) = 2\ln x$.

(vi) Simplify as far as possible

$$\left(\frac{1}{\cos\beta} + \tan\beta\right)\left(\frac{1}{\cos\beta} - \tan\beta\right).$$

B.4. Consider a particle with mass m moving in a potential $U(x)$, given by

$$U(x) = U_0 \sin^2(\alpha x),$$

where U_0 is a positive constant with dimensions of *energy*, and α is a positive constant with dimensions of *length*.

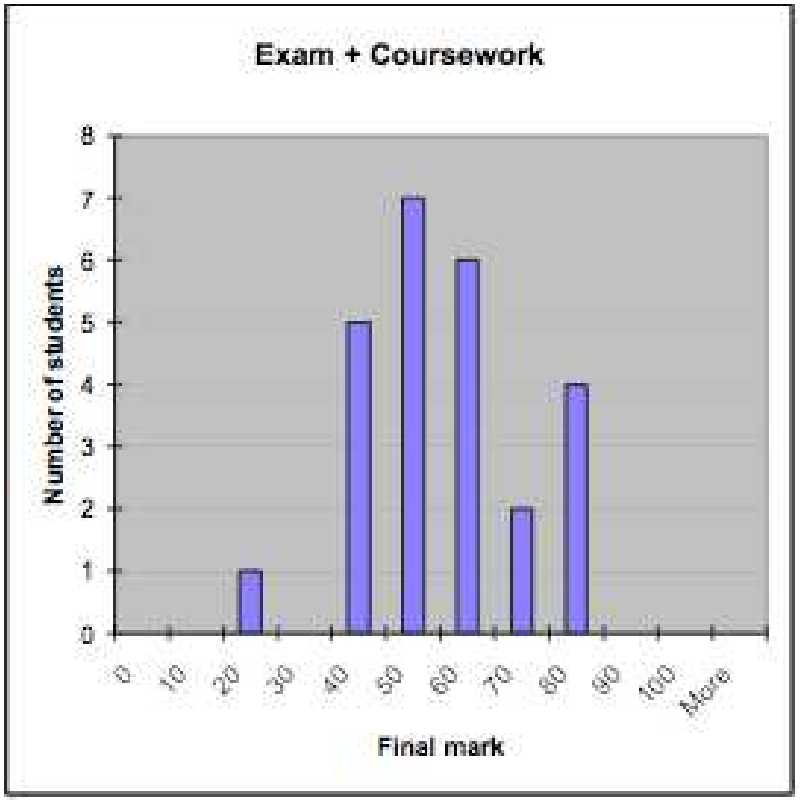
(i) Find the first three non-zero terms in the series expansion of $U(x)$ near $x = 0$. **[5]**

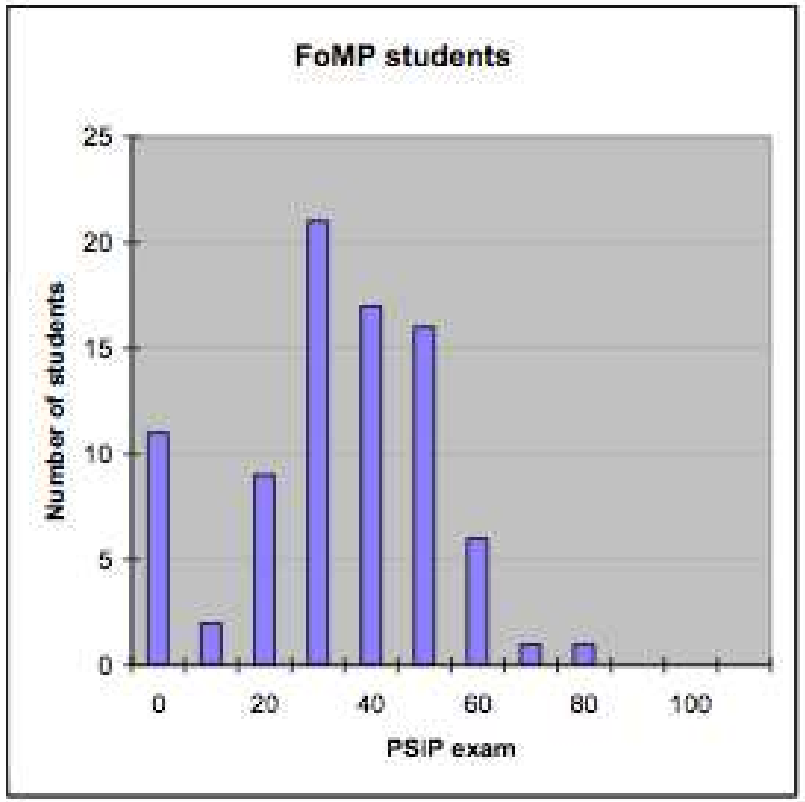
(ii) Find the values of x for which the potential $U(x)$ vanishes, and sketch $U(x)$ for $-\pi \leq x \leq \pi$. **[5]**

(iii) Add to your sketch in part (ii) three approximations to $U(x)$ obtained by retaining one, two, and three terms in the expansion found in part (i). Explain which of these would be adequate to describe small oscillations of the particle about $x = 0$, and write down the resulting classical equation of motion for the particle. **[5]**

(iv) Find the period of these small oscillations by comparing the approximate equation of motion found in part (iii) with the equation of motion for a mass on a stretched spring. **[5]**

Treatment


Exam + Coursework
Number of students
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60
70
80
90
100
More
Final mark


FoMP students
Number of students
0
5
10
15
20
25
0
20
40
60
80
100
PSIP exam

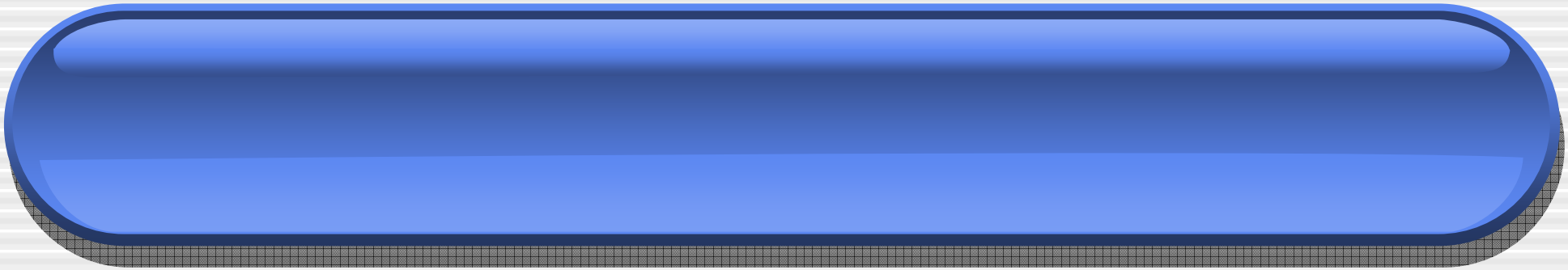
Student views

"At the time… I hated it…."

"…but afterwards… I can *do* things much quicker. The basics are there. You're not fumbling around trying to figure out stuff you should have learned before…."

Issues

- *Expert* TAs needed
- Scale-up looks beneficial, but expensive
- ‘Problem solving’ ?
- ‘Service teaching’



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